DISSIPATION EFFECTS WITH PULSATIONS OF GAS BUBBLES IN VISCOELASTIC POLYMERIC LIQUIDS

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An investigation was made of the free vibrations of gas bubbles in viscoelastic polymeric liquids, taking account of all dissipation mechanisms (rheological, acoustical, and thermal). Along with the effects of shear viscoelasticity [1], account is taken of relaxation phenomena with volumetric deformation of the medium. The dynamics of gas bubbles in incompressible viscoelastic media without taking account of heat dissipation was investigated in [2-4].

The study of dissipation effects with pulsations of gas bubbles in liquids is of interest for a description of the behavior of bubbles of mixtures under dynamic conditions [5]. It is well known [6] that the damping of the vibrations of gas inclusions in a viscous liquid is due to the following main reasons: heat dissipation as a result of heat transfer between the gas contained in the bubble and the surrounding medium; acoustical dissipation due to losses for the emission of sound by the oscillating bubbles; viscous dissipation, connected with the irreversible character of processes of the transfer of momentum in the medium and with localization in the case of an incompressible liquid near the gas–liquid interface. Within the framework of a linear approximation, an analysis of the above-listed effects for the case of a Newtonian liquid was made in [7], where the additivity of the above mechanisms of dissipation with small values of the total decrement of the damping of the vibrations of the bubble is shown. The solution of an analogous problem in the case of the pulsations of bubbles in polymeric liquids makes it necessary to take account of additional factors, above all the rheological special characteristics of the medium, connected with the appearance of phenomena of viscoelasticity. We note that the effects of viscoelasticity can approximately be taken into consideration through a dynamic boundary condition at the surface of the bubble, neglecting the accompanying effect of rheology and compressibility. An analogous method was used, for example in [7-9], taking account of the compressibility of a Newtonian liquid in a quasiacoustical approximation [10]. However, using such an approach it is not possible to evaluate the absorption of the sound emitted by an oscillating bubble, which is of independent interest (with the solution of the external hydrodynamic problem for a bubble in [7-9] the liquid is assumed to be ideal). In view of this, an investigation is made below of the dynamics of a bubble in a compressible viscoelastic liquid within the framework of a combined analysis of the rheology and the compressibility; here, the solution is first found to the problem of the emission of sound by an oscillating bubble, taking account of the rheology of the medium.

Limiting ourselves in what follows to the framework of a linear approximation, for the deviator part of the tensor of the stresses in the liquid we take the three-constant Oldroyd equation[1], used for a description of the mechanical behavior of a number of dilute polymer solutions,

$$\begin{gathered}\tau_{ij}+\lambda_1\dot{\tau}_{ij}=2\eta_0(s_{ij}+\lambda_2\dot{s}_{ij}),\ \lambda_1\geqslant\lambda_2\geqslant 0,\ \tau_{ij}=\sigma_{ij}+p\delta_{ij},\\ s_{ij}=e_{ij}-(1/3)e_{\alpha\alpha}\delta_{ij},\ p=-(1/3)\sigma_{\alpha\alpha},\\ e_{ij}=(1/2)(v_{i,j}+v_{j,i}),\end{gathered} \tag{1}$$

where τ_{ij} and s_{ij} are the deviators of the tensor of the stresses and the tensor of the deformation rates, respectively; p is the mechanical pressure; v_i are the components of the velocity vector; λ_1 and λ_2 are the times of the relaxation and the retardation, respectively; η_0 is the viscosity. For the spherical part of the stress tensor we take the equation [11]

$$p+\tau_1\dot{p}=-K_0(u+\tau_2\dot{u}),\ \tau_2\geqslant\tau_1\geqslant 0, \tag{2}$$

where u is the relative volumetric deformation; K_0 is the module of the volumetric elasticity; τ_1 and τ_2 are the relaxation and retardation times of the volumetric deformation, respectively. Equation (2) describes the volumetric deformation with the presence of one internal relaxation process of arbitrary nature in the medium [12].

We denote quantities relating to the gas and the liquid respectively by the subscripts 1 and 2. For the density ρ_2 and the pressure p_2 in the liquid we take

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$$\rho_2 = \rho_{20}(1 + s_2(r,\ t)),\ p_2 = p_{20} + f_2(r,\ t), \tag{3}$$

where ρ_{20} and p_{20} are equilibrium values; s_2 and f_2 are small perturbations; r is the radial coordinate of the spherical system of coordinates r, θ, φ, with its origin at the center of the bubble. The radial component of the velocity in the liquid is denoted by $q_2(r, t)$. Taking account of the spherical symmetry of the flow induced by the radial vibrations of the bubble, we have from (1)-(3)

$$\begin{gathered}\tau_{rr} + \lambda_1\dot{\tau}_{rr} = 2\eta_0(s_{rr} + \lambda_2\dot{s}_{rr}),\ \tau_{\varphi\varphi} + \lambda_1\dot{\tau}_{\varphi\varphi} = \\ = 2\eta_0(s_{\varphi\varphi} + \lambda_2\dot{s}_{\varphi\varphi}),\ s_{rr} = (2/3)(\partial q_2/\partial r - q_2/r), \\ s_{\varphi\varphi} = -(1/2)s_{rr},\ \tau_{\theta\theta} = \tau_{\varphi\varphi},\ f_2 + \tau_1\dot{f}_2 = K_0(s_2 + \tau_2\dot{s}_2).\end{gathered} \tag{4}$$

The linearized equations of motion and continuity have the form

$$\rho_{20}\frac{\partial q_2}{\partial t} = -\frac{\partial f_2}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{2(\tau_{rr} - \tau_{\varphi\varphi})}{r}; \tag{5}$$

$$\frac{\partial s_2}{\partial t} + r^{-2}\frac{\partial}{\partial r}(r^2 q_2) = 0. \tag{6}$$

We postulate that the gas bubble has the equilibrium radius R_0 and performs small free damping vibrations with the complex frequency h:

$$R = R_0 + \Delta R,\ \Delta R = \delta e^{ht}. \tag{7}$$

Let

$$\partial s_2/\partial t = hs_2,\ \partial f_2/\partial t = hf_2,\ \partial q_2/\partial t = hq_2,\ \partial\tau_{rr}/\partial t = h\tau_{rr},$$
$$\partial\tau_{\varphi\varphi}/\partial t = h\tau_{\varphi\varphi}\ .$$

Then, from system (4)-(6) we have for q_2

$$q_2 = m^{-2}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q_2)\right),$$
$$m^2 = \rho_{20}h\left[\frac{K_0(1+\tau_2 h)}{h(1+\tau_1 h)} + \frac{4\eta_0(1+\lambda_2 h)}{3(1+\lambda_1 h)}\right]^{-1}.$$

Setting $q_2 = \varphi_r$, for the potential φ we obtain the Helmholtz equation with the complex parameter

$$\nabla^2\varphi - m^2\varphi = 0. \tag{8}$$

The solution of Eq. (8) corresponding to diverging waves has the form

$$r\varphi = A\exp(ht - mr).$$

For the complex rate c of propagation of spherical sound waves, emitted by a bubble oscillating in a compressible viscoelastic liquid, we have the dispersion equation

$$c^2 = \frac{K_0(1+\tau_2 h)}{\rho_{20}(1+\tau_1 h)} + \frac{4\eta_0 h(1+\lambda_2 h)}{3\rho_{20}(1+\lambda_1 h)}.$$

We determine the constant A from the kinematic condition at the surface of the bubble taking account of (7). We obtain

$$A = -\frac{cR_0^2\eta\delta}{c + hR_0}e^{hR_0/c}.$$

For the perturbation of the pressure in the liquid we now have

$$f_2 = -\frac{K_0(1+\tau_2 h)Ah^2}{c^2 h(1+\tau_1 h)r}\exp\left(ht - \frac{h}{c}r\right). \tag{9}$$

From (9) we find an expression for the pressure at the surface of the bubble

$$p_2(R,\ t) = p_{20} + \frac{K_0 h^2 R_0(1+\tau_2 h)}{c(1+\tau_1 h)(c + hR_0)}\Delta R. \tag{10}$$

In the case of a Newtonian liquid ($\tau_1 = \tau_2 = \lambda_1 = \lambda_2 = 0$), the dynamic correction to the pressure at the surface of a pulsating bubble assumes the form

$$\begin{gathered}f_2(R,\ t) = \rho_{20}h^2 R_0(c_0/c)^2(1 + hR_0/c)^{-1}\Delta R, \\ c^2 = c_0^2\left(1 + \frac{4}{3}\eta_0 h\rho_{20}^{-1}c_0^{-2}\right),\quad c_0 = (K_0/\rho_{20})^{1/2}.\end{gathered} \tag{11}$$

We note that formula (11) differs from the corresponding expression for f_2, found according to the linearized quasiacoustical theory [7-9], by the addition of terms taking account of the combined effect of the viscosity and compressibility of the liquid. For not too small bubbles in a slightly viscous and slightly compressible liquid the contribution of these terms is small.

The dynamic boundary condition at the surface of the bubble has the form

$$p_2(R,\ t) = p_1(R,\ t) - 2\sigma R^{-1} + \tau_{rr}\ (R,\ t), \tag{12}$$

where $p_1(R, t)$ is the pressure in the gas at the surface of the bubble; σ is the coefficient of surface tension. Determining τ_{rr} (R, t) and substituting relationship (10) into (12), with an exactness up to linear quantities we find

$$p_1(R,\ t) = p_{20} + \frac{2\sigma}{R_0} + \left\{\frac{K_0 h^2 R_0 (1+\tau_2 h)}{c(1+\tau_1 h)(c+hR_0)} - \frac{2\sigma}{R_0^2} + \frac{4\eta_0(1+\lambda_2 h) R_0^2 h}{3(1+\lambda_1 h)(1+hR_0/c)}\left(\frac{3}{R_0^3} + \frac{3h}{cR_0^2} + \frac{h^2}{c^2 R_0}\right)\right\}\Delta R. \tag{13}$$

We postulate that the bubble contains an ideal gas with the equation of state

$$p_1 = \rho_1 T_1 c_V(\gamma - 1),\ \ \gamma = c_p/c_V,$$

where T_1 is the temperature; c_p and c_V are the specific heat capacities with constant pressure and volume. We set

$$T_1 = T_0(1 + \theta(r,\ t)),\ \ p_1 = p_{10}(1 + s_1(r,\ t) + \theta(r,\ t)),$$
$$\rho_1 = \rho_{10}(1 + s_1(r,\ t)),\ \ p_{10} = \rho_{10} T_0 c_V(\gamma - 1),$$

where θ and s_1 are small perturbations of the temperature and the density.

The linearized system of equations of the conservation of mass, momentum, and energy for the gas in the bubble is written in the form

$$\begin{gathered}\frac{\partial s_1}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q_1) = 0,\\ \rho_{10}\frac{\partial q_1}{\partial t} = -p_{10}\left(\frac{\partial s_1}{\partial r} + \frac{\partial\theta}{\partial r}\right),\\ \partial\theta/\partial t = D_1\nabla^2\theta + (\gamma - 1)\partial s_1/\partial t,\ \ D_1 = \chi/\rho_{10} c_V,\end{gathered} \tag{14}$$

where q_1 is the radial component of the velocity in the gas; χ is the coefficient of thermal conductivity.

We set

$$\partial q_1/\partial t = hq_1,\ \ \partial s_1/\partial t = hs_1,\ \ \partial\theta/\partial t = h\theta.$$

Solving [7] the system of equations (14), determining $p_1(R, t)$, and taking into consideration that the pressure p_{10} in the bubble satisfies the equilibrium equation, from (13) we obtain a transcendental equation for h

$$\begin{gathered}h^2 = F(h),\quad F(h) = -\frac{c^2(1+\tau_1 h)(1+hR_0 c^{-1})}{K_0 R_0^2(1+\tau_2 h)}\times\\ \times\left[G - \frac{2\sigma}{R_0} + \frac{4\eta_0(1+\lambda_2 h)R_0^3 h}{3(1+\lambda_1 h)(1+hR_0 c^{-1})}\left(\frac{3}{R_0^3} + \frac{3h}{cR_0^2} + \frac{h^2}{c^2 R_0}\right)\right],\\ G = p_{10} R_0^2 D_1 (l_2 - l_1)\{(hl_1^{-1} - D_1)\left[(R_0 l_1^{1/2})\operatorname{cth} R_0 l_1^{1/2} - 1\right] -\\ -(hl_2^{-1} - D_1)\left[(R_0 l_2^{1/2})\operatorname{cth} R_0 l_2^{1/2} - 1\right]\}^{-1},\\ p_{10}\rho_{10}^{-1} D_1 h^{-1} l^2 - (\gamma p_{10}\rho_{10}^{-1} + hD_1)\, l + h^2 = 0.\end{gathered} \tag{15}$$

With determination of the solution of the system (14) for the temperature T_1, the boundary condition $T_1(R) = T_0 = \text{const}$, adopted in [6-9, 13], was used. A need to consider the external problem of thermal conductivity can arise only in the presence of phase transitions at the interface or with strong constriction of the gas in the bubble, which is excluded by the statement of the problem under consideration.

Equation (15) makes it possible to calculate the frequency and the damping of the vibrations of a bubble taking account of the combined effect of thermal, acoustical, and rheological dissipation. We also give an equation for h, characterizing only one definite form of dissipation. The thermal deformation is characterized by the equation

$$h_1^2 = -(\rho_{20} R_0^2)^{-1}(G(h_1) - 2\sigma R_0^{-1}). \tag{16}$$

Taking account of acoustical dissipation leads to the equation

$$\begin{gathered}h_2^2 = -(1 + h_2 R_0 c^{-1})(\rho_{20} R_0^2)^{-1}(3kp_{10} - 2\sigma R_0^{-1}),\\ c^2 = \rho_{20}^{-1} K_0 (1 + \tau_2 h_2)(1 + \tau_1 h_2)^{-1},\end{gathered} \tag{17}$$

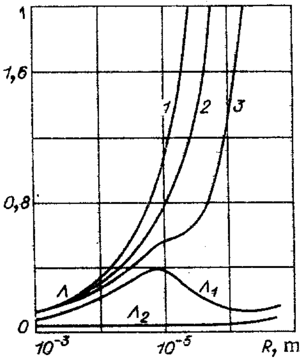

Fig. 1

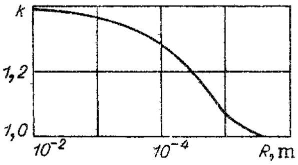

Fig. 2

where k is the index of the polytrope in the equation for the gas in a bubble

$$p_1(R, t) = p_{10}(1 - 3kR_0^{-1}\Delta R).$$

Finally, taking account only of rheological dissipation leads to the following exact solution for h [2]:

$$\mathrm{Re}\{h_3\} = -(A + B)/2 - a/3, \quad \mathrm{Im}\{h_3\} = \sqrt{3}(A - B)/2, \tag{18}$$

$$A = (-U/2 + \sqrt{Q})^{1/3}, \quad B = (-U/2 - \sqrt{Q})^{1/3},$$

$$Q = (V/3)^3 + (U/2)^2, \quad U = 2(a/3)^3 - ab/3 + g,$$

$$V = -a^2/3 + b, \quad a = \lambda_1^{-1} + 2\alpha\lambda_2\lambda_1^{-1},$$

$$b = \beta + 2\alpha\lambda_1^{-1}, \quad g = \beta\lambda_1^{-1}, \quad \alpha = 2\eta_0(\rho_{20}R_0^2)^{-1},$$

$$\beta = 3k\left[p_{20} + 2\sigma R_0^{-1}(1 - (3k)^{-1})\right](\rho_{20}R_0^2)^{-1}.$$

Let us determine the total logarithmic decrement of the damping Λ and the frequency ω of the vibrations of the bubble using the formulas

$$\Lambda = -2\pi\mathrm{Re}\{h\}\mathrm{Im}^{-1}\{h\}, \quad \omega = (2\pi)^{-1}\mathrm{Im}\{h\},$$

and denote by Λ_1, Λ_2, and Λ_3 the thermal, acoustical, and rheological decrements of the damping, respectively. The absorption at a wavelength of sound $\varkappa$, emitted by an oscillating bubble, is determined by the relationship

$$\varkappa = 2\pi(\mathrm{Im}\{c\}\mathrm{Re}^{-1}\{c\})/(1 - \mathrm{Im}\{c\}\mathrm{Re}\{h\}\,\mathrm{Re}^{-1}\{c\}\mathrm{Im}^{-1}\{h\}).$$

The values of h, h_1-h_3 were found numerically from relationships (15)-(18) in a BÉSM-4 digital computer. The thermophysical parameters of the air in the bubble, and the pressure, the density, and the surface tension in the liquid were taken as follows: $T_0 = 293°K$; $\gamma = 1.4$; $\chi = 0.0257$ J/(m · sec · deg); $c_p = 10^3$ J/(kg · deg); $p_{20} = 10^5$ N/m^2; $\rho_{20} = 10^3$ kg/m^3; $\sigma = 0.05$ N/m. The values of the rheological and acoustical parameters were selected in accordance with the experimental data of [14-16] from the range characteristic for a number of polymer solutions.

The solutions of the transcendental equations (15)-(17) were determined by an iteration method. The value of the index of the polytrope k ($1 \le k \le 1.4$) in finding h_2 and h_3 was determined by linear interpolation from the condition of the consistency of the natural frequencies of the vibrations, calculated using Eqs. (17), (18), and (15), respectively. For a verification of the correctness of the calculation, a calculation was first made of the decrements of the damping of air bubbles in water at a temperature of 20°C. The results obtained were compared with the data of [7].

Some characteristic curves for viscoelastic liquids are given in Figs. 1-4. Curve 1 corresponds to a Newtonian liquid ($\lambda_1 = \lambda_2 = \tau_1 = \tau_2 = 0$); curves 2, 3 correspond to a liquid with shear viscoelasticity ($\tau_1 = \tau_2 = 0$) with $\lambda_1 = 10^{-2}$ sec and $\lambda_2 = 5 \cdot 10^{-3}$, $2 \cdot 10^{-3}$ sec, respectively; curves 4, 5 correspond to a liquid with shear and volumetric viscoelasticity with $\lambda_1 = 10^{-2}$ sec, $\lambda_2 = 2 \cdot 10^{-3}$ sec, $\tau_1 = 10^{-10}$ sec, and $\tau_2 = 1.5 \cdot 10^{-10}$, $2 \cdot 10^{-10}$ sec, respectively. For all the curves $\eta_0 = 10^{-2}$ kg/(m · sec); $K_0 = 2 \cdot 10^9$ N/m^2.

It follows from the curves that taking account of shear viscoelasticity leads to a decrease in the total decrement of the damping Λ (Fig. 1). This effect appears for medium and small gas inclusions and is reinforced with a decrease in the radius of the bubbles. Calculations showed that, with a rise in the viscosity of the liquid, the effect of shear viscoelasticity rises; here, there is an increase in the characteristic dimension of the bubbles, the vibrations of which are subject to viscoelastic effects.

The dependence of the effective index of the polytrope k on the radius of the bubble is shown in Fig. 2. It can be seen that, for large bubbles, the value of k is close to γ, while, for small values ($R_0 < 10^{-5}$ m), it coin-

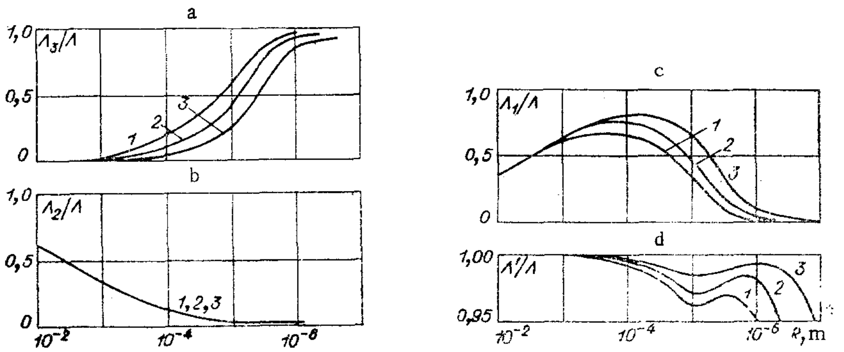

Fig. 3

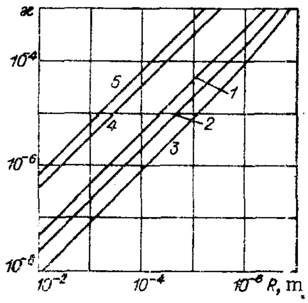

Fig. 4

cides with the isothermal value k=1. We note that the form of the curve $k=k(R_0)$ does not change with a variation of the rheological parameters of the medium and is determined only by thermal effects [7].

As calculations showed, the natural frequency of the vibrations of bubbles taking account of shear viscoelasticity rises only slightly; however, this is observed only for very small values of $R_0 \sim 10^{-6}$ m, for which k=1. The volumetric viscoelasticity has no significant effect on the damping and the frequency of the vibrations of gas inclusions (curves 3-5 in Figs. 1,3 coincide).

Figure 3 illustrates the relative contribution to the total decrement of the damping Λ of the rheological, acoustical, and thermal dissipation mechanisms ($\Lambda' = \Lambda_1 + \Lambda_2 + \Lambda_3$). It can be seen that the effect of shear viscoelasticity leads to an increase in the range of bubble sizes; the damping of the bubbles is determined predominantly by heat dissipation and to a decrease in the relative role of rheological dissipation, which continues to dominate for small inclusions. We note that the relative contribution of acoustical dissipation changes only insignificantly when the viscoelasticity is taken into consideration (curves 1-3 on the scale of Fig. 3b come together into one curve). This is connected with the fact that the rheological dissipation remains considerable in the case of medium and small bubbles, for which the relative role of acoustical dissipation is not great. As a result of this, with an investigation of the influence of viscoelastic effects on the damping of such bubbles it is permissible, with a determined degree of approximation, to assume that the liquid is incompressible. Calculations show that the error arising from such an assumption in determination of the value of Λ for bubbles with $R_0 < 10^{-4}$ m does not exceed 10%, and is lowered with an increase in the viscosity of the liquid or a decrease in the radius of the bubbles.

As follows from Fig. 3d, the rheological, acoustical, and thermal mechanisms of dissipation in a viscoelastic liquid, as in a Newtonian liquid, are additive in a wide range of bubble sizes. Slight deviations from additivity are observed only for small bubbles. For such inclusions, superposed effects appear, due to the combined contributions of different dissipation mechanisms, which are not taken into consideration in Λ'. As a result of this, the value of Λ' is found to be somewhat lower than the total decrement of the damping Λ. We note that, in a viscoelastic liquid, the deviation of the value of Λ' from Λ is less than in a Newtonian liquid. In view of this, with an investigation of the damping of bubbles in a viscoelastic liquid, a separate investigation can be made of different dissipation mechanisms (including rheological [3]), with a smaller error than in a Newtonian liquid.

Figure 4 characterizes the absorption of sound, emitted by an oscillating bubble. It can be seen that the volumetric and shear viscoelasticities exert an opposite effect on the damping of the sound waves and lead, correspondingly, to a rise or a decrease in the value of $\varkappa$. Under these circumstances, the absorption of sound in a liquid with volumetric and shear viscoelasticity is found to be greater than in a Newtonian liquid with the same viscosity. This result is in agreement with the experimental data of [17], pointing to a decrease in the cavitational noise in a liquid with polymer additives.

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